

Ad-soyad :

Numara :

C E V A P A N A H T A R I

Lineer Cebir II Arasnav Sorulari

30.04.2020

1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ 3 & 6 \end{bmatrix}$ matrisleri için (20 p)

a) $AB = ?$

b) $BA = ?$

c) $A^t = ?$

d) $A-2B = ?$

e) $B^t = ?$

2) Elementer operasyonlar yardımıyla $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ -4 & 3 & 5 \end{bmatrix}$ matrisinin tersini bulunuz (25 p).

3) $\begin{cases} -6x - 3y + z = -5 \\ 3x + y - z = 1 \\ 5x + 2y - z = 4 \end{cases}$ lineer denklem sistemini çözünüz (25 p).

4) $A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ lineer dönüşümü $A(x, y, z) = (x + y, y - z, x + 2z)$ şeklinde tanımlanıyor (25 p).

a) A lineer dönüşümünün çekirdeğini bulunuz.

b) $\text{rank} A = ?$

c) A lineer dönüşümünü matris formunda ifade ediniz.

C E V A P L A R

1) a) $AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 2 \cdot 3 + 3 \cdot 3 & 1 \cdot 2 + 2 \cdot (-4) + 3 \cdot 6 \\ 4 \cdot (-1) + 5 \cdot 3 + 6 \cdot 3 & 4 \cdot 2 + 5 \cdot (-4) + 6 \cdot 6 \end{bmatrix}$
 $= \begin{bmatrix} 14 & 12 \\ 29 & 24 \end{bmatrix}$

b) $BA = \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -1 \cdot 1 + 2 \cdot 4 & -1 \cdot 2 + 2 \cdot 5 & -1 \cdot 3 + 2 \cdot 6 \\ 3 \cdot 1 + (-4) \cdot 4 & 3 \cdot 2 + (-4) \cdot 5 & 3 \cdot 3 + (-4) \cdot 6 \\ 3 \cdot 1 + 6 \cdot 4 & 3 \cdot 2 + 6 \cdot 5 & 3 \cdot 3 + 6 \cdot 6 \end{bmatrix}$
 $= \begin{bmatrix} 7 & 8 & 9 \\ -13 & -14 & -11 \\ 27 & 36 & 45 \end{bmatrix}$

$$c) A^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

d) Tipleri farklı olup $A=2B$ teminli değildir

$$e) B^t = \begin{bmatrix} -1 & 3 & 3 \\ 2 & -4 & 6 \end{bmatrix}$$

$$2) \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -4 & 3 & 5 & 0 & 0 & 1 \end{array} \right] \xrightarrow{S_1+S_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 7 & 9 & 4 & 0 & 1 \end{array} \right] \xrightarrow{S_3/7}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 9/7 & 4/7 & 0 & 1/7 \end{array} \right] \xrightarrow[S_2+S_3]{S_2+S_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 16/7 & 4/7 & 1 & 1/7 \end{array} \right] \xrightarrow[-S_2]{\frac{7}{16} S_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1/4 & 7/16 & 1/16 \end{array} \right] \xrightarrow[-2S_3+S_1]{S_3+S_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/8 & -1/8 \\ 0 & 1 & 0 & 1/4 & -3/16 & 1/16 \\ 0 & 0 & 1 & 1/4 & 7/16 & 1/16 \end{array} \right] \xrightarrow{A^{-1}}$$

3) Sistemin ilaveleli katsayılar matrisi

$$\left[\begin{array}{ccc|c} -6 & -3 & 1 & -5 \\ 3 & 1 & -1 & 1 \\ 5 & 2 & -1 & 4 \end{array} \right] \xrightarrow{S_1/-6} \left[\begin{array}{ccc|c} 1 & 1/2 & -1/6 & 5/6 \\ 3 & 1 & -1 & 1 \\ 5 & 2 & -1 & 4 \end{array} \right] \xrightarrow[-5S_1+S_3]{-3S_1+S_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & -1/6 & 5/6 \\ 0 & -1/2 & -1/2 & -3/2 \\ 0 & -1/2 & -1/6 & -1/6 \end{array} \right] \xrightarrow[-S_2+S_3]{S_2+S_1} \left[\begin{array}{ccc|c} 1 & 0 & -2/3 & -2/3 \\ 0 & -1/2 & -1/2 & -3/2 \\ 0 & 0 & 1/3 & 4/3 \end{array} \right] \xrightarrow[\frac{3}{2}S_2+S_2]{2S_3+S_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1/2 & 0 & 1/2 \\ 0 & 0 & 1/3 & 4/3 \end{array} \right] \Rightarrow x=2, -\frac{1}{2}y=\frac{1}{2}, \frac{1}{3}z=\frac{4}{3}$$

$$\Rightarrow x=2, y=-1, z=4$$

$$4) a) \text{Ker } A = \{ (x, y, z) \in \mathbb{R}^3 : A(x, y, z) = (0, 0, 0) \}$$

$$= \{ (x, y, z) \in \mathbb{R}^3 : (x+y, y-z, x+2z) = (0, 0, 0) \}$$

$$\begin{cases} x+y=0 \\ y-z=0 \\ x+2z=0 \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \xrightarrow{s_3 \leftrightarrow s_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{-s_1+s_3}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{-s_2+s_3} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{\substack{2s_3+s_1 \\ -s_3+s_2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{-s_3}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow x=0, y=0, z=0$$

$$\Rightarrow \text{Ker } A = \{ (0, 0, 0) \}$$

$$b) \dim \mathbb{R}^3 = \text{rank } A + \underbrace{\text{sifirlik } A}_0$$

$$\Rightarrow \text{rank } A = 3$$

$$c) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y-z \\ x+2z \end{bmatrix}$$